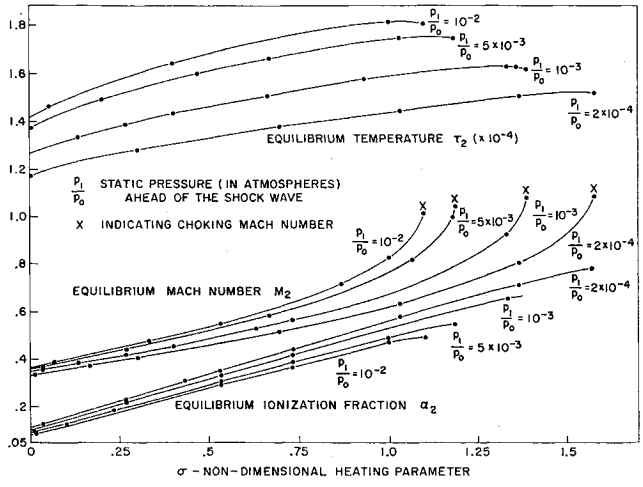

Fig. 1 Temperature, Mach number, and ionization fraction at equilibrium behind shock wave for argon, $M_1 = 15$.

potential of the gas in degrees Kelvin. Other notations in the forementioned equations have their usual meaning.

In the present analysis, q is assumed to be explicitly independent of the derivatives of n, u, A, T, and α with respect to x. An example is the case of Ohmic heating. This implies that the condition for thermal choking does not depend explicitly on q.

Two Mach numbers may be introduced:

$$M = u(3m/5kT)^{1/2} \tag{6}$$

$$M_p = u\{3m/5k(1 + \alpha)T\}^{1/2} \tag{7}$$

where m is the mass of the neutral gas molecule, M refers to the speed of sound of the neutral gas, whereas $\{5k(1 + \alpha)T/3m\}^{1/2}$ is essentially the sound speed of the ionized gas as a whole.

From the Saha equation, Eq. (4), it is found

$$\frac{2 - \alpha}{\alpha(1 - \alpha)}\frac{d\alpha}{dx} = \frac{1}{u}\frac{du}{dx} + \left(1.5 + \frac{T_{\text{ion}}}{T}\right)\frac{1}{T}\frac{dT}{dx} \tag{8}$$

By using Eqs. (1–3 and 8),

$$\left\{M_p^2[1 + (\gamma - 1)(1.5 + t_i)F(t_i,\alpha,\chi)] - \left[1 + \frac{\gamma - 1}{\gamma}(2.5 + t_i)F(t_i,\alpha,\chi)\right]\right\}\frac{1}{u}\frac{du}{dx} = \left\{1 + \frac{\gamma - 1}{\gamma}(1.5 + t_i)F(t_i,\alpha,\chi)\right\}\frac{1}{A}\frac{dA}{dx} - \frac{q}{\gamma R(1 + \alpha)mnuT} \tag{9}$$

where $t_i = T_{\text{ion}}/T$, $\chi = 2(1 - \alpha)/(2 - \alpha)$, and

$$F(t_i,\alpha,\chi) = \frac{t_i\alpha\chi}{2 + (2 + 1.5\chi + \chi t_i)\alpha}$$

Equation (9) shows that in a straight duct (A = const), an addition of heat to a subsonic flow ($M_p \ll 1$) always causes the Mach number M_p to rise. A limit will be reached, however, when the factor of (du/udx) in Eq. (9) vanishes. Hence, choking of the flow occurs when

$$M_p^2\{1 + (\gamma - 1)(1.5 + t_i)F(t_i,\alpha,\chi)\} = 1 + [(\gamma - 1)/\gamma](2.5 + t_i)F(t_i,\alpha,\chi) \tag{10}$$

This expression reduces to the well-known formula $M = 1$, when $\alpha = 0$. It reduces also to the simple form

$$M_p = 1 \tag{11}$$

when $\alpha = 1$ (fully-ionized gases). In general, the value of M_p depends on the ionization fraction and the function T_{ion}/T and is found to be generally close to, but not exactly equal to, one. For $\gamma = \frac{5}{3}$, M_p^2 is found to be generally less than one and is equal to one only for $\alpha = 0$ and 1. Equation (10) suggests the possibility of defining a Mach number, which becomes one at choking for $0 \leq \alpha \leq 1$. Its physical meaning is, however, obscure.

Consider the thermal choking of the flow behind a normal shock wave. Let subscript 1 refer to conditions of the neutral monatomic gas ($\alpha = 0$) ahead of the shock wave and subscript 2 to the equilibrium conditions of the partially-ionized gas behind the shock wave. The hydrodynamic conservation equations are (with $\gamma = \frac{5}{3}$)

$$n_1u_1 = n_2u_2 \tag{12}$$

$$mn_1u_1^2 + n_1kT_1 = mn_2u_2^2 + n_2(1 + \alpha_2)kT_2 \tag{13}$$

$$\tfrac{1}{2}mu_1^2 + \tfrac{5}{2}kT_1 + H = \tfrac{1}{2}mu_2^2 + (1 + \alpha_2)\tfrac{5}{2}kT_2 + \alpha_2kT_{\text{ion}} \tag{14}$$

where H is the rate of energy density addition to the gas. In nondimensional form, Eqs. (12) and (13) can be written as

$$\tfrac{5}{2}M_1^2 + 1 = [\tfrac{5}{3}M_2^2 + (1 + \alpha_2)](M_1/M_2)\tau_2^{1/2} \tag{15}$$

$$\tfrac{1}{3}M_1^2 + 1 + (2M_1^2/3)\sigma = [\tfrac{1}{3}M_2^2 + (1 + \alpha_2)]\tau_2 + \tfrac{2}{3}\alpha_2\tau_i \tag{16}$$

where $\tau_2 = T_2/T_1$, $\tau_i = T_{\text{ion}}/T_1$, and $\sigma = H/mu_1^2$ is a nondimensional heating parameter.

Calculations have been carried out for M_2, τ_2, and α_2 by using Eqs. (15, 16, and 4). The results for argon ($T_{\text{ion}} = 182{,}000°\text{K}$) are shown in Fig. 1 for various values of p_1/p_0 and σ. The choking Mach numbers M_2 also are shown.

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Thermal and Electrical Properties of Thin-Film Resistance Gages Used for Heat Transfer Measurement

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Calibration data are given for the variation of the thermal and electrical properties of a thin-film heat transfer gage in the range from 70 to 1000°F. The gage comprises a platinum-alloy film mounted on a pyrex substrate. Calibration is accomplished by the electrical pulsing technique.

AN experimental study of base heating of rocket vehicles[1] necessitated the development of a thin-film resistance thermometer gage operable at steady-state temperatures

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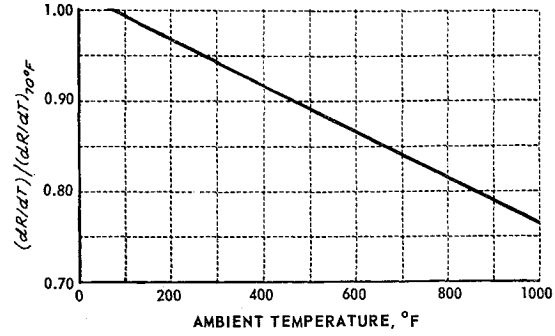

Fig. 1 Variation of thin film sensitivity with temperature.

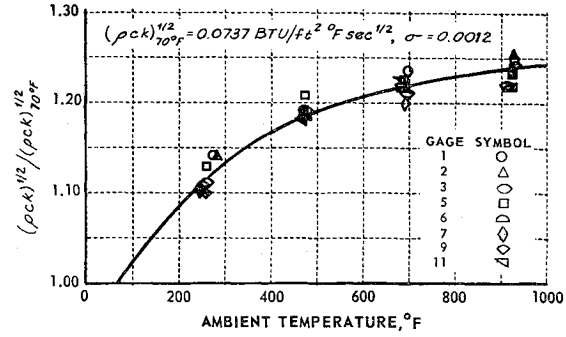

Fig. 2 Variation of $(\rho ck)^{1/2}$ with temperature #7740 Pyrex.

to 1000°F. To convert gage measurements to surface heat flux, it was necessary to make an accurate calibration in this temperature range of the electrical and thermal properties of the gage, which comprised a thin, platinum-alloy film deposited on a pyrex substrate.[2] These calibration results represent an extension in temperature range of similar measurements made by Skinner[3] who employed conventionally constructed gages that have an upper, steady-state temperature limit of approximately 400°F.

The calibration of the thermal properties of the pyrex substrate was accomplished using the electrical pulsing technique described by Skinner.[3, 4] Certain refinements were added to the circuit arrangement, and the method of reading the recorded data traces was modified.[2]

At a temperature of 70°F, the mean value of $(\rho ck)^{1/2}$ (where ρ is density, c is specific heat, and k is thermal conductivity) for pyrex was measured as 0.0737 Btu ft^{-2} °F^{-1} sec$^{-1/2}$ with a standard deviation from the mean of 0.0012. This value is in close agreement with 0.0743 ± 5% reported by Skinner and 0.074 ± 16% reported by Vidal.[5] These data indicate a high degree of uniformity in the properties of pyrex specimens and further justify the widespread use of pyrex as a gage substrate.

To measure the variation of $(\rho ck)^{1/2}$ with temperature in the interval to 1000°F by the technique indicated, it was necessary to account for the nonlinearity in the resistance-temperature characteristic of the thin platinum films.† Thus for a unit change in temperature, the change in film resistance would be expected to be function of the ambient temperature. Gage resistance-temperature measurements were taken in the range to 1000°F, and a least-squares function fitted to these data. A second-degree equation in temperature provided a satisfactory fit. By normalizing the change in gage sensitivity (i.e., the incremental change in resistance for an incremental change in temperature dR/dT), very little numerical deviation between gages was found. Accordingly, the gage data were pooled into one composite relation that is plotted in Fig. 1 and described by Eq. (1)

$$(dR/dT)/(dR/dT)_{70°F} = 1.018 - 2.59 \times 10^{-4}T \quad (1)$$

where T is the ambient temperature in degrees Fahrenheit.

The data of Fig. 1 contrast with the results given in the pertinent references cited,[3-5] where dR/dT was taken as a constant to temperatures of 400°F. It is probable that the restricted temperature range in which these investigations were made (400°F was the approximate maximum steady-state value attained) obscured the temperature effect reported here.

Applying corrections for changes in gage sensitivity with temperature, the normalized variation of $(\rho ck)^{1/2}$ with temperature was evaluated. The results, summarized in Fig. 2, represent a smaller effect of temperature and exhibit less scatter than has been reported previously. For example, in the range where a comparison of results is possible, Skinner[3] and Hartunian[6] determined the mean $(\rho ck)^{1/2}$ ratio at 400°F as 1.375 and 1.54 (extrapolated), respectively, compared with a mean of 1.165 given in Fig. 2. One factor that accounts partially for the smaller thermal dependence is the correction applied for the decrease in gage sensitivity with temperature.

The data in Figs. 1 and 2 are important to the interpretation of measurements taken with thin-film resistance thermometers in applications at elevated steady-state temperatures and when large amplitude transient temperatures are involved.

† Liquid Bright Platinum solution #05-X, Hanovia Liquid Gold Division, East Newark, N. J.

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Hypervelocity Stagnation Point Heat Transfer in a Carbon Dioxide Atmosphere

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Nomenclature

P_w = stagnation point wall pressure
P_3 = pressure behind bow shock of model
R_N = model nose radius
V_3 = flow velocity behind bow shock of model
V_f = simulated flight velocity
ρ_3 = density behind bow shock of model

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